ONTARIO DEPARTMENT OF EDUCATION

CURRICULUM P1, J1

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372.7
OSIDE/M

INTERIM REVISION MATHEMATICS

1966

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INTERIM REVISION/MATHEMATICS GRADES 1 TO 6

INTRODUCTION

There are two main divisions in this course. The first section, Principles and Topics, consists of a statement of aims and three lists of concepts. The second section, entitled Suggestions, outlines the kinds of attitudes, methods and materials necessary for achieving the aims of the course.

Each of the three lists of topics in the first section contains an outline roughly illustrating sequence of difficulty, and each commences with topics suitable for the first year of the Primary Division. Teachers will realize the need to interweave topics from each of the three lists at times. The possibility of developing concepts from more than one of the lists, at the same time, is inherent in most activities of children. For example, when children work with water and various-sized containers, ideas about fractional number concepts, liquid measurement and conservation of volume may be developed.

No specific assignment of topics to grade levels is desirable. The placement of any topic at any particular age level of children is arbitrary. Recent research and experimentation show that the background of experience of each child, and the method by which a topic is approached, are more valid requisites of readiness than the assignment of a topic to a grade through tradition.

The following is an interpretation of the sequence of dots, solid lines and dashes after each topic in the three sections of the syllabus:

. . . . . informal and incidental experiences

_____ planned and directed activities

----- learning refined, extended and enriched.

Any exactnesss in relating the above interpretation of the dots, lines and dashes to grade levels is to be avoided. The dots, lines and dashes are merely suggestive and are intended to allow for the wide variety of topic placement shown to be successful in several modern mathematics programs.

Sequential presentation of topics is only necessary in a broad sense, as is indicated in each of the three lists of topics. Children may work in groups at apparently unrelated tasks in mathematics, and still develop mastery of basic principles.

During the six years of the Primary and Junior Divisions, children should develop some mastery of the concepts indicated in this course. The ability and interest of each child will determine the degree of mastery, as has always been the case.

The Suggestions in the second section of the course are by no means comprehensive. A wealth of aids of many kinds is available or aids may be developed by teachers, by various committees and groups, or obtained from commercial sources. Handbooks, manuals and teachers' guides can aid teachers in planning learning experiences for children. The production of Curriculum Guides would be a valuable in-service activity for teachers.

I PRINCIPLES AND TOPICS GRADES 1 TO 6

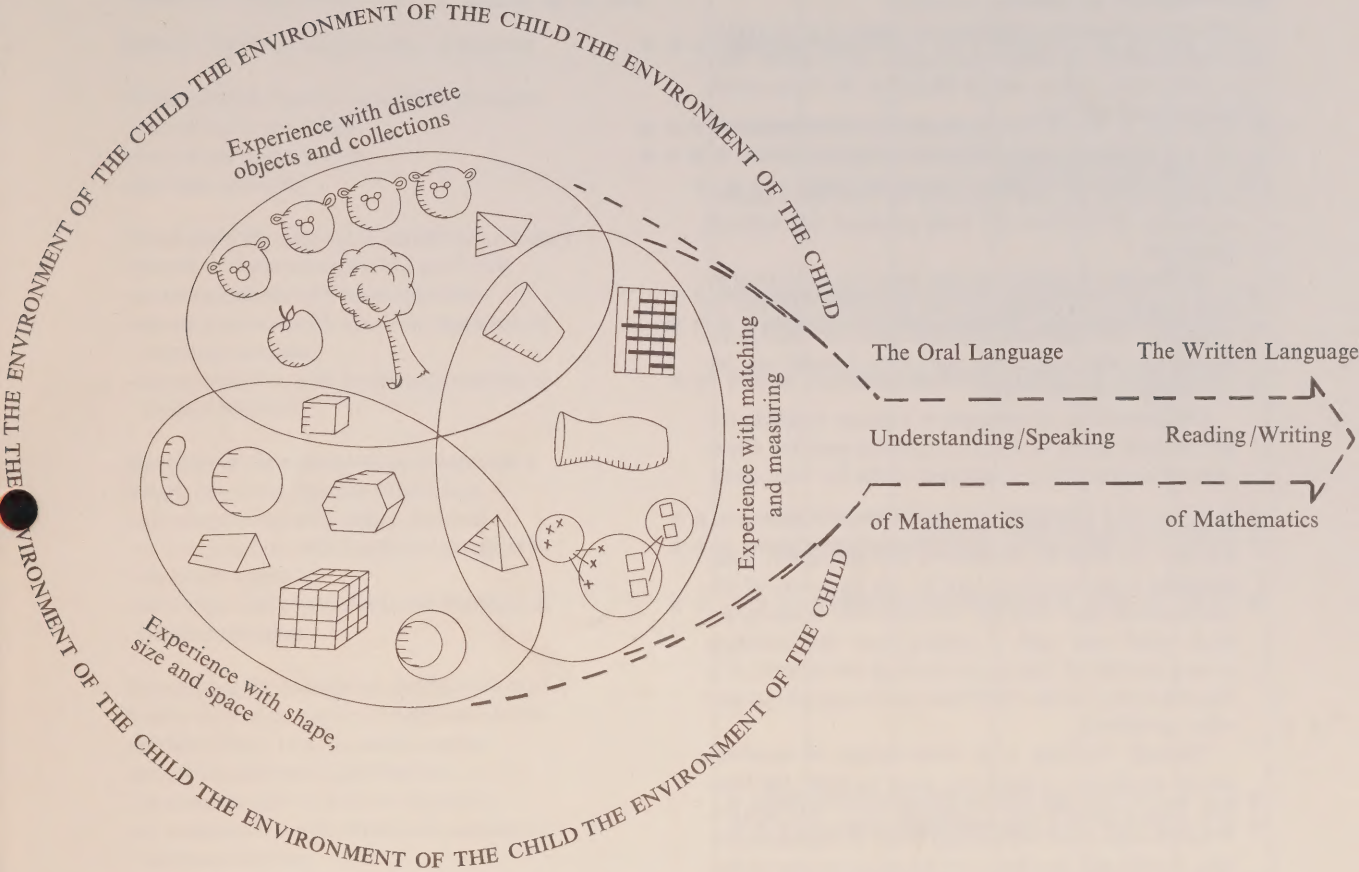
AIMS

1 Since the previous course was prepared, research in learning by children and experimentation in the teaching of mathematics have led to the adoption of several new concepts in the elementary school curriculum. These may be summarized as follows:
   a) Children should be introduced to mathematics rather than arithmetic which is just one aspect of the subject.
   b) Mathematics may be regarded as a medium of communication (See page 4). The parallel between the development of the language of mathematics and the native language of the child has become widely recognized in current teaching practices in the Kindergarten and the Primary Division.
   c) Mathematics in the early grades should be treated as an environmental study using an inductive approach and hence may merge into science, geography, art, crafts, etc. It should develop its concepts and its applications from real situations.
   d) A variety of procedures in solving mathematical problems should be accepted. It is desirable that pupils devise their own methods of attack.
   e) There will be wide variations in the times at which individual children arrive at a stage of mastery of even the fundamental facts of arithmetic.
   f) Speed of recall is not a necessary concomitant of accuracy and accuracy is not a necessary consequence of repetitive "drill".

2 The development of mathematical concepts in young children is best achieved through an inductive process. Experiences with real objects and events in the child's environment should be the starting points for the growth of mathematical ideas. The abstracting of mathematical concepts from situations where children are at play or at work requires perceptive teaching. For example, when children start the "marble and skipping" season, the redistribution of marbles among the boys after each recess can provide data for bar graphs from which a variety of number facts and relationships may be deduced.

Teachers who know the basic mathematical concepts outlined in this course will be able to recognize the worthwhile discoveries made by children. Much discussion among children as they work together to solve practical problems should precede any reporting of mathematical ideas in symbolic form.

As in language, so in mathematics there should be recognition of a general sequence of development. The first stage is that of experience with concrete things and comprehension of the patterns and relationships thus discovered. As a child is involved in many experiences of many kinds, he will gradually gain sufficient security to describe orally the mathematics he knows. Undue urging of a child at this stage will create attitudes of insecurity and self-doubt which can become dislike of the subject. Time must be allowed for the child in the primary grades to have experiences and to comprehend for himself.


THE ENVIRONMENT OF THE CHILD THE ENVIRONMENT OF THE CHILD THE ENVIRONMENT OF THE CHILD
Experience with discrete objects and collections
Experience with matching and measuring
Experience with shape, size and space
The Oral Language
The Written Language
Understanding/Speaking
Reading/Writing
of Mathematics
of Mathematics
THE ENVIRONMENT OF THE CHILD THE ENVIRONMENT OF THE CHILD THE ENVIRONMENT OF THE CHILD

MATHEMATICS AS A MEDIUM OF COMMUNICATION

Mathematics may be thought of as a special language which enables man to communicate ideas about quantity and shape and relationships. Its grammar is logic and its vocabulary is a wide variety of symbols.

In Grades 1 to 6 the basic teaching method should be inductive, with emphasis upon the discovery of principles by each child for himself. A wide variety of real experiences with real materials is necessary.

The three divisions of this course ought to be developed concurrently. A single experience of a child may serve to develop ideas which illustrate the close interrelationship of all three.

1 Number and Operations

A child needs an understanding of, and some facility in, the use of numbers for both practical and cultural purposes.

Arithmetic should be taught from the beginning in such a way as to emphasize the general properties of numbers. This algebraic approach aids the child as he begins to order and comprehend for himself our increasingly complex world.

Understanding of concepts is a prime requisite for all children. There is also a continuing need for a reasonable degree of computational skills for each child.

2 Measurement and Relationships

As the link between arithmetic and geometry, measurement is an important tool for the discovery of the mathematics that is in the environment. It has practical social uses such as telling time and handling money and, in an experience-centred curriculum, it is also necessary for the performance of experiments and other activities.

Through working with relationships of numbers and of quantities, a child can learn to think for himself. This is particularly evident in work with graphs. When he discovers pattern and order in his surroundings, a child will develop confidence in his own ability and a liking for the subject.

3 Shapes and Space

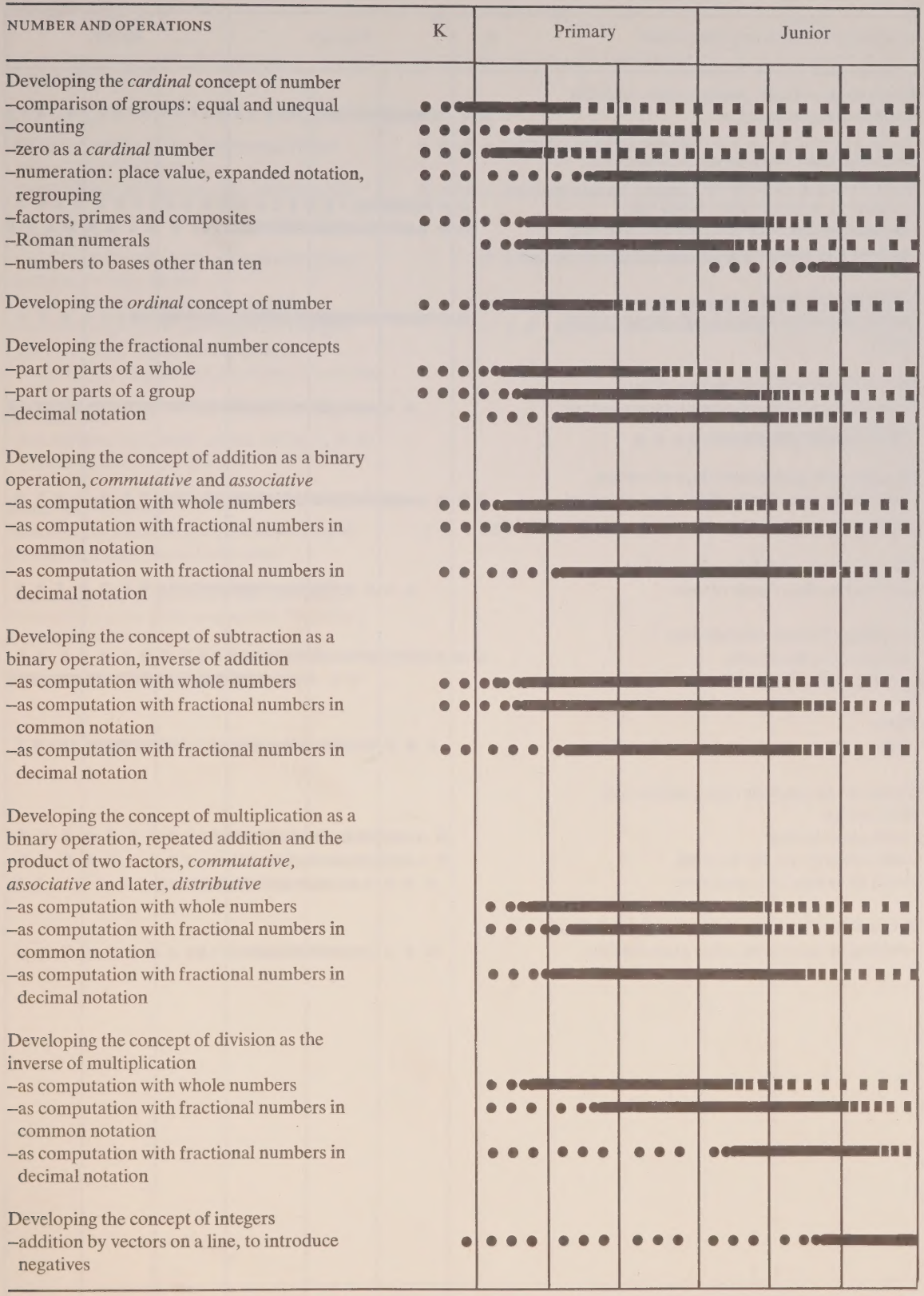
The physical environment of the child is composed of shapes and space. The relevance of geometrical ideas, as the child begins to order and comprehend this complex world for himself, is obvious.

An intuitive approach is best, with children discovering properties of shapes, symmetry, similarity, then dimensions, in a progression of experiences. *No introduction of formal proofs in geometry is recommended.*

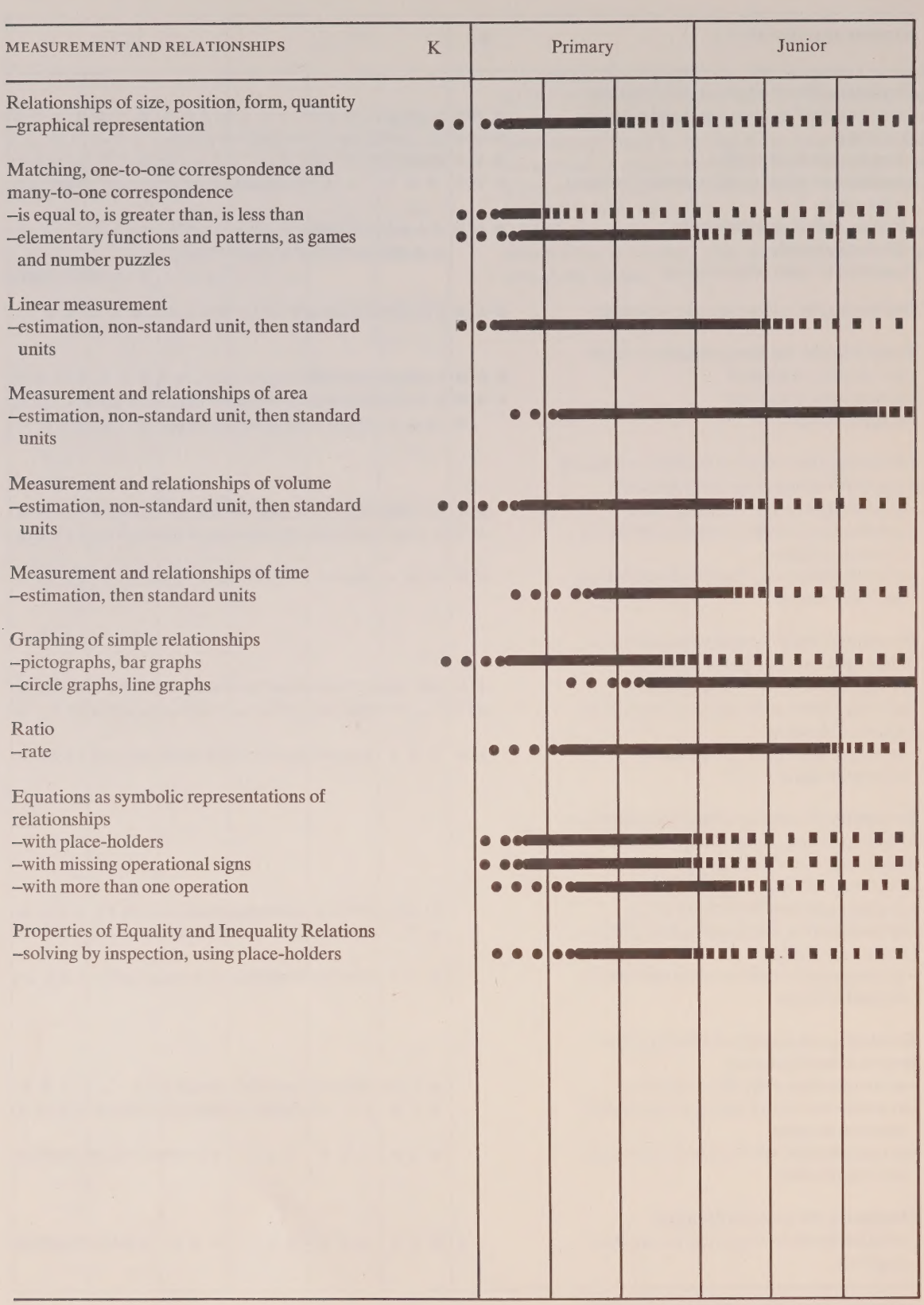
The possibility of inter-relationships among topics listed under Number and Operations and under Measurement and Relationships should be explored.

Research and feasibility studies are needed concerning the introduction of three- and two-dimensional geometrical ideas to children from the age of six to 12. Teachers are urged to be especially observant as children respond to experiences with three-dimensional materials, and to record and inform others of the kinds of discoveries made by pupils.

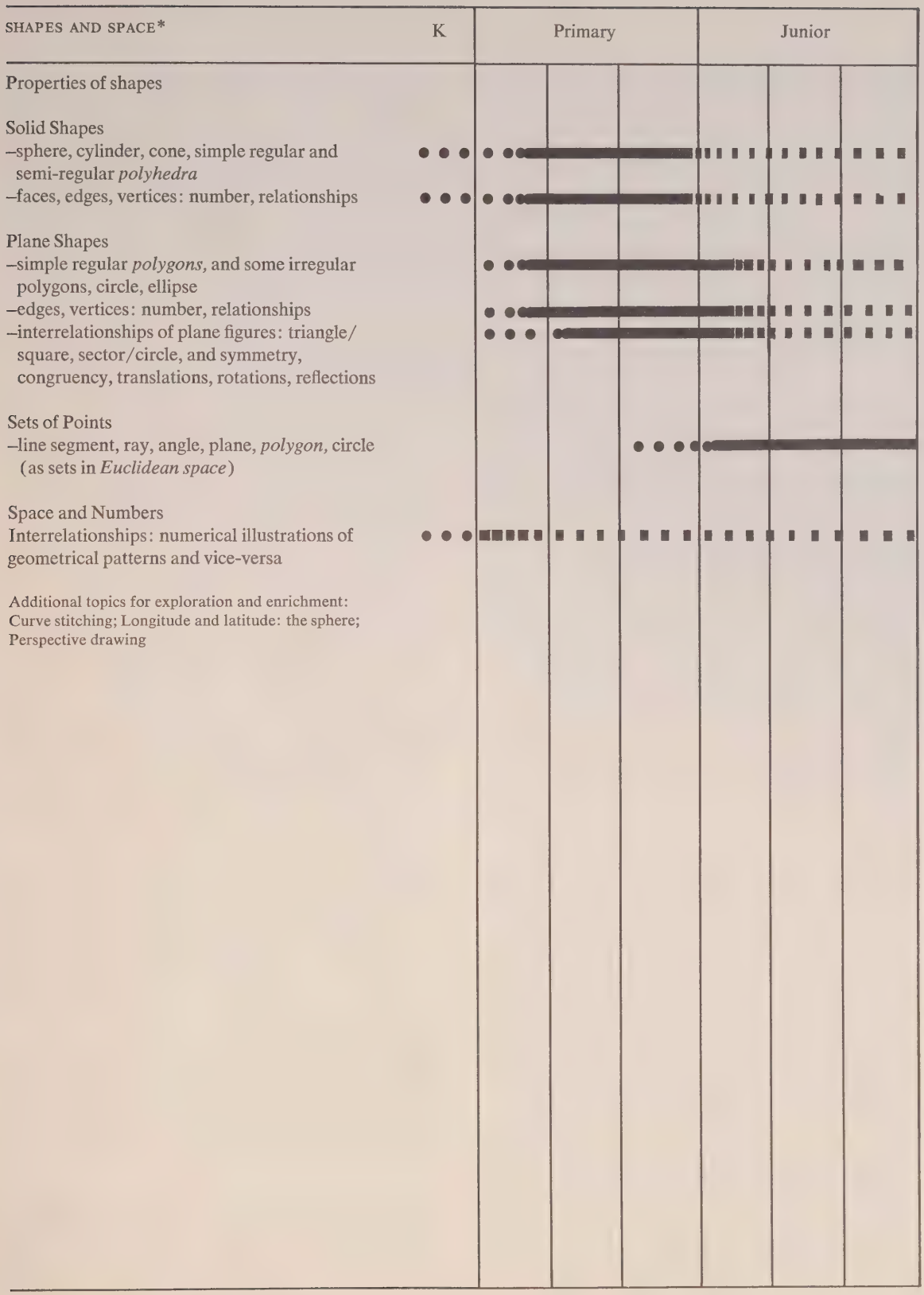
Similar studies would be valuable in the area of interrelationships of numbers and space; or indeed, in any area of the course.



Additional topics for exploration and enrichment: Bookkeeping; Games and Patterns; History of Mathematics; Simple statistical presentation of data.



Additional topics for exploration and enrichment:
Co-ordinate systems; Metric system of measurement;
Scale Drawings; Vectors



*This section is an integral part of the course, but it should be treated experimentally, with different outcomes expected from different children and different classes.

II SUGGESTIONS

MATERIALS AND METHODS

Experiences in which the ideas of mathematics are abstracted from the environment of the child are best for the achieving of sound and lasting learning. The joy of discovery is the privilege of every child. The perceptive teacher makes a point of commending the 'moments of triumph' which result.

The learning of mathematics in elementary schools is a process of development *from the concrete* or real world, *to the semi-abstract,* and then *to the abstract* world of number, shape and relationships. Each teaching material must be used in its proper role. For example, such a sequence may be seen where the number of children bringing lunch to school each day for a week is recorded pictorially in a graph. Then a variety of number phrases is formulated from the relationships arrived at through class discussion.

1 Discovery techniques and problem-centred activities cannot be the only methods employed in classrooms, but a judicious balance of the presentation, the discussion, the reference-research and the experience-activity types of lessons should be maintained by the teacher.
2 It is anticipated that television will increasingly provide assistance to teachers of mathematics, as of most other subjects. Care must be exercised to ensure that a balanced variety of learning situations is maintained for the children but the potential of this medium of communication can scarcely be over-estimated.
3 When wide variations in the work of children in a classroom are expected, then the use of class sets of a single textbook is not desirable. Such a practice is inconsistent with the encouragement of experiences within the real environment of the child. A textbook should be employed as a reference and guide by a child, or as an application step – the culmination of some learning that has taken place elsewhere. Furthermore, textbooks should not be thought of as having a particular grade designation. If a child or a group of children are capable of progressing into the next book of a series, or a book at a higher level, this should be arranged. Similarly, children whose progress is slower than others should use books which provide for reasonable success on their part.
4 Small pamphlets and the laboratory or kit of spirally graded materials based on the idea of self-directed activity are increasingly useful materials. Programmed books are also useful in meeting the needs of individual pupils.
5 There is room, too, for commercially-prepared structural materials in mathematics in addition to films, filmstrips and other audio-visual media. Manipulative devices of many kinds should be provided to enable children to add tactile sensation to visual and auditory experiences.
6 A great many materials from the immediate environment of the children may be used in activities from which mathematics may be abstracted. For example, compartmented candy boxes can hold groups of marbles, geo-boards may be made with finishing nails and coloured rubber bands, a set of textbooks spread out on a table top may be used to illustrate measurement of area in arbitrary units.
7 The use of instruction cards, prepared by the teacher and used by pupils as guides for self-directed activities, is encouraged, as is the setting up of mathematical laboratories much like the library corner equipped with all manner of measuring devices and other apparatus.

Careful selection should be made of all materials. A basic criterion should be that a "tool" in the hand of each pupil is far more valuable than any didactic device employed by the teacher. It is more worthwhile to invest

in a class set of concrete materials that enable each pupil to discover principles for himself, than to purchase demonstration equipment which is simply an adjunct to the lecture method.

Careful evaluation of textbooks should be a continuing responsibility of teachers. Discarding obsolete books is an important aspect of the selection of materials for use in the classroom.

Emphasis should be given to methods of recording results in a wide variety of ways – pictographs, bar graphs, charts and tables, Venn diagrams, experience chart stories, etc.

Opportunities for the abstracting of mathematical ideas from the activities of other subjects of the curriculum should not be lost. If mathematics appears to merge into science, geography, art and other subjects, so much the better.

STANDARDS OF ACHIEVEMENT

The syllabus in mathematics suggested in Section I is partially sequential – the development of skills in children from five to approximately twelve years of age must be somewhat orderly, and too, the structure of the subject governs the sequence of topics to a certain extent. But it should not be forgotten that experience is an undivided whole for children. No two children, or classes, or schools, should be required to follow the same route through this syllabus.

If the syllabus is thought of as only one possible natural progression in the thinking of a child, rather than a list of topics to be imposed on a class, then the learning which children do ought not to be subordinated to the examination.

1 *The teacher is in the best position to evaluate the progress of his own pupils.* Examinations, and the more preferable short-term (or single concept) test, should be based on the teaching that has taken place in the classroom, and should be differentiated for groups or individuals that have been following different routes within the class.
2 The understanding of a concept, or related cluster of concepts, might be evaluated whenever there is a need to estimate formally the progress of a pupil or a group of pupils. The development of tests, or better still, patterns for tests, might be done by committees of teachers.
3 In all formal testing of mathematics, care must be exercised to ensure that principles, rather than only procedural techniques, are examined. For example, knowing when to subtract is at least as important as knowing how to perform one of the standard algorithms for subtraction. Accuracy is, of course, a basic requisite.
4 The teacher who knows his pupils and who is competent in the subject is a better judge of pupil mastery than any test or examination. The comparing of a child's achievement on a test with an external or group standard is a questionable practice if used to make value judgments rather than diagnostic decisions.
5 Reporting to parents is best done in terms of the child's progress in mastery of the concepts of the course, rather than comparisons with his presumed ability or comparisons with a "class standard". Some reference to external standards – such as provincial or system-wide norms – may be desirable occasionally to inform parents of the relative rate of progress of a child.

ADAPTING TO CHANGE

Teachers have a professional responsibility to maintain competence in the changing subject matter of mathematics, as well as to work towards improved methodology. Some of the following suggestions may be of use in this regard.

1 A short but representative list of books and films is included at the end of this section, for use by interested teachers and in-service groups.
2 One or more staff members of a school could undertake to become a resource person in mathematics, by such means as:
   – membership in the Ontario Association of Teachers of Mathematics (Ontario Educational Association)
   – membership in the National Council of Teachers of Mathematics (U.S.A.)
   – subscription to the Ontario Mathematics Gazette
   – subscription to *The Arithmetic Teacher*
   – attendance at university and Department of Education courses in mathematics.
3 The vitality of mathematical ideas at every grade level in a school may be maintained by staff discussions, by mathematical clubs among the pupils, by co-operative teaching, and by the encouragement of experimentation.
4 It is highly desirable that local groups of teachers take responsibility for adaptation of this course. Some aspects of such adaptations might be:
   – Interweaving of the concepts of number, relationships and shapes into short units of work outlines.
   – Scheduling of work in accordance with local patterns of classroom organization (if such scheduling is deemed necessary): grades, units, or alternative schemes
   – Collating and distributing suggestions for the teaching of various topics of the course as an aid to teachers
   – Formulating short single-concept tests to help a teacher evaluate a child's progress in mastery of the individual concepts of the syllabus.

REFERENCES FOR TEACHERS

The Schools Council, Curriculum Bulletin No 1, *Mathematics in Primary Schools,* Her Majesty's Stationery Office, York House, Kingsway, London W.C. 2, England. Available in Canada through the British Information Services, 200 University Avenue, Toronto 1, Ontario.

Ontario Mathematics Commission, *Mathematics, Report of the Committee Considering the Mathematics Programme (K to 6),* Ontario Curriculum Institute, 102 Bloor Street West, Toronto 5, Ontario. (after September 1, 1966)

National Council of Teachers of Mathematics, *The Growth of Mathematical Ideas, K-12,* The Yearbook, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington, 20036, D.C. U.S.A.

Educational Services Incorporated, *Goals for School Mathematics,* (Houghton Mifflin Company) Thomas Nelson & Sons (Canada) Ltd., 81 Curlew Drive, Don Mills, Ontario.

The Schools Council, *Maths Alive,* 16 mm. film available after September 1, 1966, from Audio-Visual Branch, Ontario Department of Education, 559 Jarvis Street, Toronto 5.